FAR BEYOND

MAT122

Exponential Function

Stony Brook University

Exponent Laws

$b^x b^y = b^{x+y}$

ex. $b^3 b^4 = b^{3+4} = b^7$

$\dfrac{b^x}{b^y} = b^{x-y}$

ex. $\dfrac{b^5}{b^2} = b^{5-2} = b^3$

ex. $\dfrac{b^2}{b^5} = b^{2-5} = b^{-3}$

$\left(b^x\right)^y = b^{xy}$

ex. $\left(x^2\right)^3 = \underbrace{xx \cdot xx \cdot xx}_{\text{3 sets of 2}} = x^6 = x^{3\cdot 2}$

$(abc)^x = a^x b^x c^x$

ex. $\left(2b^2c^4\right)^5 = 2^5 \; b^{10} \; c^{20}$

$= 32 \; b^{10} \; c^{20}$

$\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

ex. $\left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}$

Express exponent as a positive number:

$b^{-x} = \dfrac{1}{b^x} \qquad \dfrac{1}{b^{-x}} = b^x$

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

ex. $\sqrt{\dfrac{16}{25}} = \dfrac{\sqrt{16}}{\sqrt{25}} = \dfrac{4}{5}$

$\sqrt[n]{b} = b^{1/n}$

$\sqrt[3]{27} = 3 \quad \therefore 27^{\frac{1}{3}} = 3$

revisit: $\dfrac{b^2}{b^5} = \dfrac{\cancel{b}\cancel{b}}{\cancel{b}\cancel{b}bbb} = \dfrac{1}{b^3}$

Exponential Functions

Compare the two functions:

Power Function: $f(x) = x^2$ versus Exponential Function: $f(x) = 2^x$

base changes and gets squared

base is constant; power changes

where a_0 represents initial condition (constant)

Exponential Function: $f(x) = a_0 b^x$ where b is a positive constant and not equal to zero and not equal to one

show why later

Introduction to Exponential Applications

The exponential function represents many situations in the natural/social sciences, economics, etc.

Often the exponent represents time.

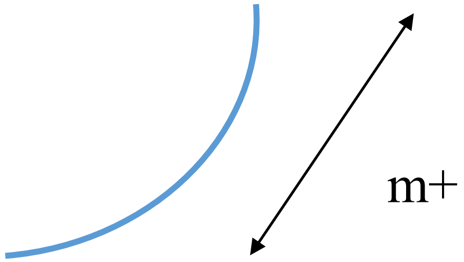
The base is the rate that function is changing.
Depending on the value of the base, over time, the result "*y*" will increase or decrease.

<u>General Shapes of Exponential Graph</u>:

curvy versions of linear function

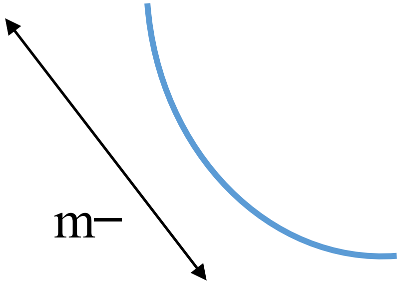
As time increases, so does resulting amount.

Exponential Growth



As time increases, resulting amount decreases.

Exponential Decay



More on Exponential Function Format

Examples of Exponential Functions:

$$f(x) = 4^x \qquad f(x) = \left(\frac{1}{2}\right)^{x-1}$$

Not Exponential Functions:

$$f(x) = x^2$$

power f

$$f(x) = (-1)^x$$

b can't be negative

$$f(x) = x^x$$

both can't be same variable

$$f(x) = 1^x$$

$$f(1) = 1^1 = 1$$
$$f(2) = 1^2 = 1$$
$$f(3) = 1^3 = 1$$
$$\vdots$$

constant f

Sketching Exponential Functions

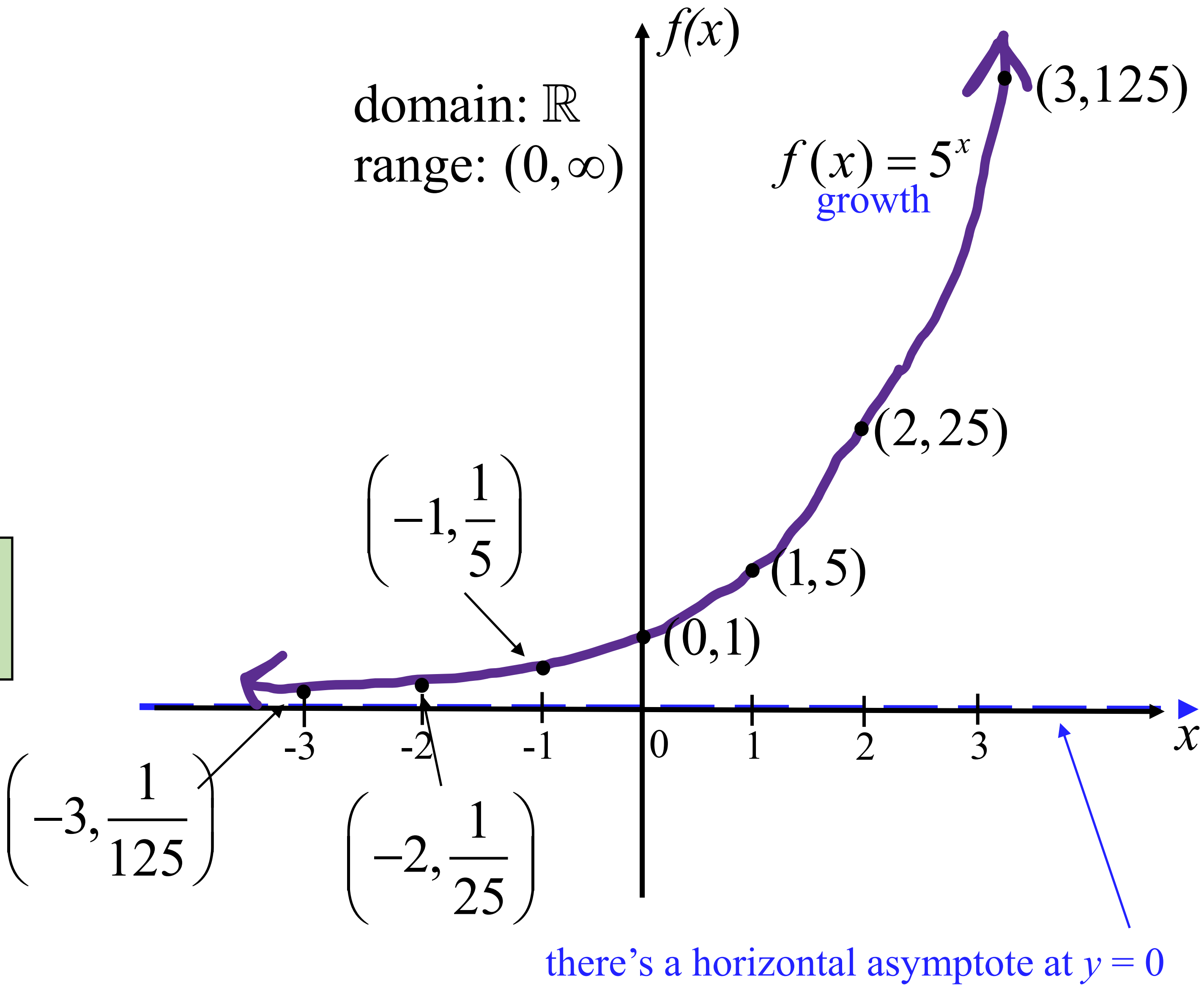
ex. Sketch $f(x) = 5^x$.

Get ordered pairs:

(use a Table of Values)

x	$y = f(x) = 5^x$
0	$f(0) = 5^0 = 1$ anything to the zero power is 1
1	$f(1) = 5^1 = 5$
2	$f(2) = 5^2 = 25$
-1	$f(-1) = 5^{-1} = \frac{1}{5^1} = \frac{1}{5}$

$$b^{-x} = \frac{1}{b^{+x}}$$



ex. Sketch $f(x) = \left(\frac{1}{5}\right)^x$.

Basic Growth Example

ex. The population of a sub-Saharan African country* had a population of 12.853 million people in 2003. Growth continued through 2015 at a rate of 3.4% every year. Determine the model of this function.

*Burkina Faso

$$P(t) = a_0 b^t$$

When $t = 0$: initial population $= 12.853 = a_0$

Base: $100\% + 3.4\% = 103.4\% \xrightarrow{decimal} 1.034$

Then population, P, after 2003 is given by: $P(t) = 12.853(1.034)^t$

ex. increase of 15% results in:

Original 100%
plus 15%

$115\% \xrightarrow{decimal} 1.15$

Follow up Q: What is the population of Burkina Faso after 2013?

$t = 0$ → year 2003

year 2013 → $t = 10$

$P(10) = 12.853(1.034)^{10}$ use calculator PEMDAS

$= 12.853(1.397)$ round to 3 places

$= 17.956$ million people in 2013

Basic Decay Example

ex. When a patient is given medication, the drug enters the bloodstream.
Over time the drug is metabolized and gradually eliminated from the body.

For ampicillin, approximately 40% is eliminated every hour.
A typical dose of ampicillin is 250mg. Determine the function of this model.

When $t = 0$: initial amount $= 250 = a_0$

Base: If 40% is being eliminated per hour then 60% remains. $\xrightarrow{decimal} 0.6$

Then quantity, Q, after t hours is given by: $Q(t) = 250(0.6)^t$

$f(t) = a_0 b^t$

Exponential Function – Special Case

Called the natural base, $e \approx 2.71828$.